$$\Delta = \pm 2t\,(2n+1). \tag{24}$$

It follows from Eq. (24) that if $n = 1$ and $t = \pm 0.03$, then $\Delta = \pm 0.18$. Thus, small oscillations of the ratio δ_c/δ_0(3%) can lead to significant deviations of the breakup radius (±18%), which corresponds to the phenomenon observed.

NOTATION

K, n, rhenological constants; ρ, density; σ, surface tension; r, current cup radius; R, maximum cup radius; r_c, critical radius for film breakup; $\bar{r} = \bar{r} = r/R$, dimensionless current radius; $\bar{r}_c = r_c/R$, dimensionless critical radius; δ_0, δ_c, actual and critical film thicknesses; δ, current thickness; R_r, ridge radius; h_0, ridge height; h, current ridge height; θ_0, limiting wetting angle; θ, current angle of tangent to ridge surface; α, angle between axis of rotation and tangent to cup surface; ω, angular velocity of rotation; q, volume liquid flow rate; v_1 and v_φ, meridional and tangential velocities; $\beta = 4v_{lm}/\omega r$, $\psi = 4v_{\varphi m}/\omega r$, dimensionless velocities; M_σ M_ω, moments of surface and centrifugal forces; M_v, moment from velocity head; p_r, pressure within ridge; p_{vm}, pressure from velocity head; $p\omega_m$, p_{pm}, pressures from centrifugal force components tangent and normal to cup surface; Δ, deviation range of breakup radius from calculated value; $\bar{r}_{max}$, $\bar{r}_{min}$, limiting deviations of breakup radius; α_c, angle of tangent to curve $\delta_c/\delta_0 = f(\bar{r})$ at critical point; t, random oscillation of ratio δ_c/δ_0.

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LAMINAR FLOW OF A VISCOUS INCOMPRESSIBLE LIQUID OVER THE SURFACE OF SOLIDS OF REVOLUTION

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The laminar flow of a viscous incompressible liquid over the surface of stationary solids of revolution is examined for the case of circular inflow of the stream.

Dispersing agents are presently used in industry [1-6] whereby a stream (jet) of viscous liquid spreads in an axisymmetric thin layer over the surface of stationary solids of revolution of various shapes. Several works [7-13] have been devoted to the study of such laminar flows. However, the results here were obtained without regard for the parameters of the inflowing stream, which leads to the appearance of quantities in the theoretical data whose values can be found only by experiment—a serious deficiency of these researches.

There are studies [14-16] which have overcome this problem.

This article examines the axisymmetric, stable thin-layer flow of a viscous, incompressible, uniform liquid over the curvilinear surface of solids of revolution in the laminar mode as a result of the inflow of an infinite circular stream, with allowance for the working parameters of the latter.

We will examine flow of the liquid in the special system of coordinates l, η, and θ. The given coordinate system is orthogonal (Fig. 1).

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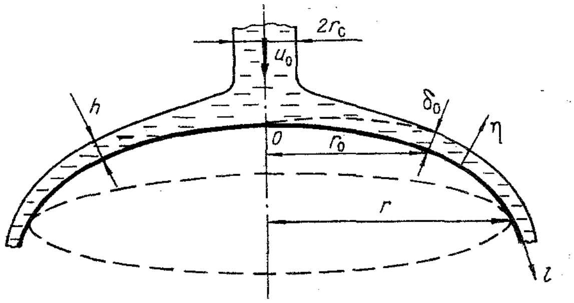

Fig. 1. Flow diagram

In solving the problem, we assume that: 1) the thickness of the layer of liquid is significantly less than its corresponding coordinate l and the radius of curvature of the surface; 2) the effect of frictional forces of the layer relative to the environment, body forces, and forces of surface tension on flow of the layer is negligible; 3) the pressure gradient in the region of film flow is small; 4) the entire thin-layer flow is divided into two regions; the first, to the line of contact of the surface of the boundary layer with the surface of the flow the second, beyond the contact line. The first region consists of the boundary layer, the thickness of which increases up to the path of the flow, and an outer layer with a velocity u_0.

Let us examine the development of the boundary layer in the first region.

Taking into consideration the above assumptions and using Slezkin's method [7] in solving the Navier–Stokes equation, the movement of the layer of viscous liquid in our case may be described by the equation

$$W_l = \nu \frac{\partial^2 v_l}{\partial \eta^2}, \tag{1}$$

where

$$W_l = \frac{1}{\delta} \int_0^\delta \left(v_l \frac{\partial v_l}{\partial l} + v_\eta \frac{\partial v_l}{\partial \eta} \right) d\eta. \tag{2}$$

In solving (1), we use the following boundary conditions:

$$v_l = 0, \; v_\eta = 0 \quad \text{at} \quad \eta = 0, \tag{3}$$

$$v_l = u_0, \; \frac{\partial v_l}{\partial \eta} = 0 \quad \text{at} \quad \eta = \delta. \tag{4}$$

Integrating (1) with boundary conditions (3) and (4), we obtain

$$W_l = \frac{2u_0 \nu}{\delta^2}, \tag{5}$$

while the equation for longitudinal velocity has the form

$$v_l = \frac{u_0}{\delta^2} (2\eta\delta - \eta^2). \tag{6}$$

Using the results obtained, we solve Eq. (2):

$$W_l = -\frac{2u_0^2}{15\delta} \left(\frac{\partial \delta}{\partial l} + \frac{\delta}{r} \frac{\partial r}{\partial l} \right). \tag{7}$$

Simultaneous solution of (5) and (7) with the condition that $l = 0$, r = 0, and $\delta = 0$ gives us

$$\delta^2 = \frac{30\nu}{u_0 r^2} \int_0^l r^2 dl. \tag{8}$$

Equation (8) is the relation for the thickness of the boundary layer formed with the flow of an axisymmetric thin layer of liquid over the curvilinear surface of solids of revolution.

Considering that the rate of flow across the boundary layer at the contact line is equal to the total rate of flow and taking Eq. (8) into account, we will write the expression for determining the coordinates of the contact line with the flow of the layer over packing of arbitrary form as follows

$$\int_0^{r_0} r^2 dl = \frac{3Q^2}{160\pi^2 \nu u_0}. \tag{9}$$

We obtain the thickness of the layer up to the contact line in the form of the sum of the thicknesses of the boundary layer and outer flow:

$$h = \frac{r_c^2}{2r} + \frac{\delta}{3}. \tag{10}$$

To find the thickness of the film beyond the contact line, we use the relation

$$h = \frac{5\pi\nu}{rQ} \int r^2 dl + \frac{C}{r}, \tag{11}$$

obtained in [11]. The contact C is found from the condition that $h = \delta$ at $r = r_0$. Then (11) takes the form

$$h = \frac{5\pi\nu}{rQ} \int r^2 dl + \frac{21 r_c^2}{32 r}. \tag{12}$$

Equations (10) and (12) represent the relations for the thickness of the layer of liquid formed in the axisymmetric spreading of a stream of liquid over the surface of solids of rotation of arbitrary shape.

Using the results obtained, it is not difficult to derive formulas for determining the thickness of the layer of liquid flowing over specific forms of solids of rotation.

Let us examine the case of plane packing. To determine the thickness of the boundary layer

$$\delta = \sqrt{\frac{10\nu r}{u_0}},$$

while the radius of the contact line

$$r_0 = 0.261 r_c \sqrt[3]{\frac{Q}{\nu r_c}}.$$

The thickness of the layer up to the contact line

$$h = \frac{r_c^2}{2r} + \sqrt{\frac{10\nu r}{9u_0}}, \tag{13}$$

and after the contact line

$$h = \frac{5\pi\nu r^2}{3Q} + \frac{21 r_c^2}{32 r}. \tag{14}$$

Equation (14) confirms the presence of a thickness minimum for the layer of liquid on the packing, discovered and examined in [7]. The radius of the line with a layer of minimum thickness is found from the formula

$$r_{\min} = 0{,}397 r_c \sqrt[3]{\frac{Q}{\nu r_c}}. \tag{15}$$

The minimum thickness may be found from the formula

$$h_{\min} = 2{,}48 r_c \sqrt[3]{\frac{\nu r_c}{Q}}. \tag{16}$$

For conical packing we will accordingly have the following

$$\delta = \sqrt{\frac{10\nu r}{u_0 \sin\alpha}}, \quad r_0 = 0.261 r_c \sqrt[3]{\frac{Q \sin\alpha}{\nu r_c}},$$

$$h_{\min} = 2.48 r_c \sqrt[3]{\frac{\nu r_c}{Q \sin\alpha}}, \quad r_{\min} = 0.397 r_c \sqrt[3]{\frac{Q \sin\alpha}{\nu r_c}}.$$

The thickness of the layer to the contact line

$$h = \frac{r_c^2}{2r} + \sqrt{\frac{10\nu r}{9u_0 \sin\alpha}}$$

and after the line

$$h = \frac{5\pi\nu r^2}{3Q\sin\alpha} + \frac{21 r_c^2}{32r}.$$

In accordance with [11-13], we will derive a relation for the thickness of the boundary layer and complete layer of liquid flowing over solids of revolution having a curvilinear generatrix. Thus, in the case of a sphere or ellipsoid, for example, we will have respectively for the boundary-layer thickness

$$\delta = \sqrt{\frac{15\nu R\left(\varphi - \frac{1}{2}\sin 2\varphi\right)}{u_0 \sin^2\varphi}},$$

$$\delta = \sqrt{\frac{30\nu}{u_0 r^2}\left\{\left[E(t,\varepsilon)\frac{2\varepsilon^2-1}{3\varepsilon^2}\right] - a^3\left[\frac{\Delta}{3}\sin t\cos t - \frac{1-\varepsilon^2}{3\varepsilon^2}F(t,\varepsilon)\right]\right\}},$$

while for the thickness of the layer of liquid up to the contact line

$$h = \sqrt{\frac{5\nu R\left(\varphi - \frac{1}{2}\sin 2\varphi\right)}{\sin^2\varphi u_0}} + \frac{r_c^2}{2R\sin\varphi}, \tag{17}$$

$$h = \sqrt{\frac{10\nu}{3u_0 r^2}\left\{\left[E(t,\varepsilon)\frac{2\varepsilon^2-1}{3\varepsilon^2}\right] - a^3\left[\frac{\Delta}{3}\sin t\cos t - \frac{1-\varepsilon^2}{3\varepsilon^2}F(t,\varepsilon)\right]\right\}} + \frac{r_c^2}{2r}$$

and after the contact line

$$h = \frac{5\pi\nu R^2}{2\sin\varphi Q}\left(\varphi - \frac{1}{2}\sin 2\varphi\right) + \frac{21 r_c}{32R\sin\varphi},$$

$$h = \frac{5\pi\nu}{rQ}\left\{E(t,\varepsilon)\frac{2\varepsilon^2-1}{3\varepsilon^2} - a^3\left[\frac{\Delta}{3}\sin t\cos t - \frac{1-\varepsilon^2}{3\varepsilon^2}F(t,\varepsilon)\right]\right\} + \frac{21 r_c^2}{32r}, \tag{18}$$

where $\Delta = \sqrt{1-\varepsilon^2\sin t}$.

The value of the functions $E(t, \varepsilon)$ and $F(t, \varepsilon)$ can be found from tables of elliptical integrals [18, 19].

Using the results obtained, we find the parameters of the fluid layer on more complicated solids of revolution: for example, for the case of flow over a plane circular torus [11].

The equation for the boundary layer may be written:

$$\delta^2 = \frac{30\nu}{u_0(r+R\sin\varphi)^2}\left[r^2R\varphi + 2rR^2(1-\cos\varphi) + \frac{R^3}{2}\left(\varphi - \frac{1}{2}\sin 2\varphi\right) + 10\nu r^3\right], \tag{19}$$

$\varphi = 0$ for the flat part of the disk and $r = r_S$ for the curvilinear part.

The thickness of the liquid layer up to the contact line may be found from (16) with allowance for (19). For the thickness of the liquid layer after the contact line, we obtain a relation of the form

$$h = \frac{5\pi\nu}{Q(r+R\sin\varphi)}\left[r^2R\varphi + 2rR^2(1-\cos\varphi) + \frac{R^3}{2}\left(\varphi - \frac{1}{2}\sin 2\varphi\right) + \frac{r^3}{3}\right] + \frac{21 r_c^2}{32(r+R\sin\varphi)}.$$

We also conducted an experimental study of the flow of a thin layer of tap water over the surface of packing. The thickness of the layer was measured by a method described in [7].

Figures 2-3 shows some of the results of the experiment and results calculated from the theoretical formulas presented here and relations in [7, 11, 14-16].

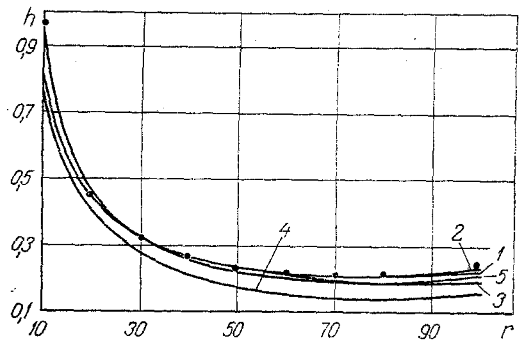

Fig. 2. Dependence of change in thickness of liquid layer h, mm, along the radius r, mm, of a flat disk: 1) from Eqs. (13), (14); 2) from [15]; 3) from [14]; 4) from [16]; 5) from [7]; the points represent empirical results; u_0 = 10 m/sec; r_c = 4 mm.

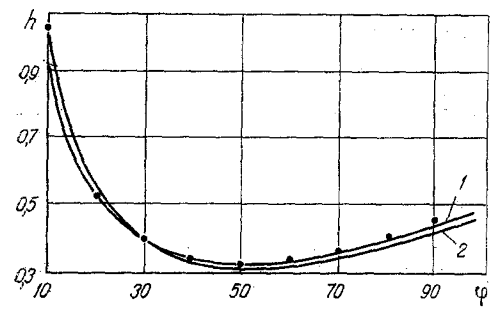

Fig. 3. Dependence of change in thickness of layer h, mm, along an arc of a spherical packing φ, deg, with a radius R = 50 mm; 1) from Eqs. (17), (18); 2) from [10]; the points represent empirical results; u_0 = 4 m/sec, r_c = 4 mm.

An analysis indicates satisfactory agreement of the results calculated with our equations and the empirical data in the film-flow region. The divergence is no greater than 8-10%. The discrepancy becomes more pronounced only in the region close to stream entry, a fact explained by the peculiarity of liquid flow in a region of impact. This permits the conclusion that the results obtained are valid in the region of film flow, beginning with $r = 2r_c$, to the zone where the layer is broken into drops. The empirical data confirms the presence of a layer-thickness minimum.

In regard to the data obtained by other authors, the best agreement is seen with [11, 14, 15]. The greatest discrepancy between our results and the results obtained in other works occurs with [16].

Thus, the results of our theoretical investigation of the axisymmetric thin-layer flow of a viscous liquid over the surface of solids of revolution of arbitrary form with the inflow of a circular stream agree satisfactorily with the empirical data.

The theoretical relations obtained make it possible to calculate the bssic parameters without recourse to additional experimental studies.

NOTATION

v_1, v_η, velocity projections; ρ, density of the liquid; ν, kinematic viscosity coefficient of liquid; R, radius of curvature of surface of the body in the flow; δ, thickness of boundary layer; u_0, rate of inflow of stream; r, current radius; Q, total flow rate; r_0, radius of contact line (circle); δ_0, thickness of layer at contact line; h, thickness of layer; r_c, radius of stream; r_{min}, radius at which layer thickness is minimal; h_{min}, minimum thickness of layer; 2α, angle at apex of conical packing; φ, current angle; ε, t, α, parameters of the ellipse; r_s, conjugate line radius of plane circular torus.

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